A hermitian square in $\mathbb{C}[z_1, \ldots, z_d, \bar{z}_1, \ldots, \bar{z}_d]$ is a polynomial of the form $\bar{h}h$, where $h \in \mathbb{C}[z_1, \ldots, z_d]$. Every hermitian square is a square in $\mathbb{R}[x_1, \ldots, x_d, y_1, \ldots, y_d]$ (where $x_j = (z_j + \bar{z}_j)/2$, $y_j = (z_j - \bar{z}_j)/2i$), but not conversely. On the other hand, hermitian squares are positive semidefinite not only under evaluations on scalars but also under evaluations on commuting matrices (or Hilbert space operators), provided we use the so called hereditary functional calculus where all the adjoints appear to the left of all the non-adjoints. In this talk (the contents of which I had the privilege of discussing with Murray Marshall in February 2015), I will present a matrix-valued version of the Hermitian Positivstellensatz due originally to Mihai Putinar as well as several applications and open problems. This talk is based on joint work with A. Grinshpan, D. Kaliuzhnyi-Verbovetskyi, and H. Woerdeman.