

Arithmetic of cuts in ordered abelian groups and of ideals over valuation rings

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Dedekind cuts are in one-to-one correspondence with their upper cut sets, which are final segments. We investigate existence, uniqueness and maximality of solutions T for equations $S_1 + T = S_2$ and inequalities $S_1 + T \subseteq S_2$ where S_1 and S_2 are final segments of ordered abelian groups. This yields information about the corresponding equalities and inequalities for cuts. We apply our results to investigate existence, uniqueness and maximality of solutions J for equations $I_1 J = I_2$ and inequalities $I_1 J \subseteq I_2$ where I_1 and I_2 are ideals of valuation rings. This enables us to compute the annihilators of quotients of the form I_1/I_2 . The results are applied in [1, 2] to compute the annihilators of Kähler differentials for Galois extensions of prime degree of valued fields.

Partial results in this direction had been obtained by Paulo Ribenboim in [4]. In my talk I will discuss in detail the progress that has been made since his work.

This is joint work with Katarzyna Kuhlmann ([3]).

References

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- [2] Cutkosky, S. D. – Kuhlmann, F.-V.: *Kähler differentials of extensions of valuation rings and deeply ramified fields*, submitted; <https://arxiv.org/abs/2306.04967>
- [3] Kuhlmann, F.-V. – Kuhlmann, K.: *Arithmetic of cuts in ordered abelian groups and of ideals over valuation rings*, submitted; arXiv:2406.10545
- [4] Ribenboim, P.: *Sur les groupes totalement ordonnés et l'arithmétique des anneaux de valuation*, Summa Brasil. Math. **4** (1958), 1–64.