Stability analysis of damped slowly rotating Timoshenko beams

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A stability analysis was performed in the problem of a rotating Timoshenko beam whose movement is controlled by the angular acceleration of the driving motor into which the beam is rigidly clamped (cf. [1]). After introducing an internal damping effect with respect to a rotation angle of a cross section area of rotating Timoshenko beam model, we obtain [2, 3] the following system of two dimensionless partial differential equations

$$\begin{cases} \ddot{w}(x,t) - w''(x,t) - \xi'(x,t) &= -u(t)(r+x), \\ \ddot{\xi}(x,t) - \gamma^2\xi''(x,t) + w'(x,t) + \xi(x,t) + \nu^2\dot{\xi}(x,t) &= u(t), \end{cases}$$

for $x \in (0,1)$ and $t > 0$, where $\dot{w} = w_t$, $w' = w_x$, $\dot{\xi} = \xi_t$ and $\xi' = \xi_x$, with boundary conditions

$$\begin{cases} w(0,t) = \xi(0,t) &= 0, \\ w'(1,t) + \xi(1,t) = \xi'(1,t) &= 0. \end{cases}$$

Here, $w(x,t)$ denotes the deflection of the center line of the beam, $\xi(x,t)$ is the rotation angle of the cross section area, ν is a damping constant and x, t stand respectively for the position and time.

Next, we show some important spectral properties of operator connected with the system. Furthermore, we show the asymptotic stability of the system under certain assumptions on the physical parameter γ^2. We find the optimal damping coefficient, maximizing the stability margin of the system.

References

[1] W. Krabs, G. M. Sklyar, *On Controllability Of Linear Vibrations*, Nova Science Publishers Inc. 2002, Huntington, NY.

[2] J. Woźniak, M. Firkowski, *Optimal damping coefficient of a slowly rotating Timoshenko beam*, Proc. SIAM Conf. Cont. Appl. 2015, pp. 81 - 84.

[3] J. Woźniak, M. Firkowski, *Optimal decay ratio of damped slowly rotating Timoshenko beams*, Z. Angew. Math. Mech. (in review).