Open problems in valuation theory of positive residue characteristic

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An example of a valued field

Take a rational function field $K(X)$ over a field K. For every $a \in K$, any rational function f in $K(X)$ may have a zero or a pole at a. If it has a zero, we take $v_a(f)$ to be the multiplicity of the zero; if it has a pole at a, we take $v_a(f)$ to be the negative of the pole order. If f has neither a pole nor a zero at a, then $v_a(f) = 0$. The set of values under v_a is the integers, and it is an ordered abelian group, called the value group. We frequently use "t" in the place of "X", and what we just defined is the t-adic valuation v_t of $K(t)$. The completion of K w.r.t. v_t is the power series field $K((t))$ with the canonical extension of v_t, again called the t-adic valuation and denoted by v_t .

Some preliminaries on valued fields

For a valued field (L, v), we denote its value group by vL. We have $v_tK(t) = v_tK((t)) = \mathbb{Z}$. The valuation ring of (L, v), denoted by $\mathcal{O}_L$, is $\{b \in L \mid v(b) \geq 0\}$. This has a unique maximal ideal $\{b \in L \mid v(b) > 0\}$, denoted by $\mathcal{M}_L$. We denote by Lv the residue field $\mathcal{O}_L/\mathcal{M}_L$. We have that $K(t)v_t = K$. The residue map $\mathcal{O}_L \to \mathcal{O}_L/\mathcal{M}_L$ can be extended to all of L by sending every element in $L \setminus \mathcal{O}_L$ to ∞. In this way, we obtain the place associated with v. For example, the place associated with v_t on $K(t)$ is obtained by sending $f(t) \in K(t)$ to $f(0)$. The valuation ring of $(K(t), v_t)$ consists of all rational functions f that do not have a pole at 0, and the residue field is equal to K.

Immediate extensions and maximal valued fields

By $(L|K,v)$ we denote an extension $L|K$ with a valuation v on L, where K is endowed with the restriction of v. In this case, there are induced embeddings of vK in vL and of Kv in Lv. The extension $(L|K,v)$ is called immediate if these embeddings are onto. A valued field (K,v) is called algebraically maximal if it does not admit nontrivial immediate algebraic extensions, and maximal if it does not admit any nontrivial immediate extensions. Every valued field admits maximal immediate extensions, and they are maximal valued fields.

Kaplansky fields

In the seminal paper

[1] Irving Kaplansky: *Maximal fields with valuations I*, Duke Math. Journ. **9** (1942), 303–321,

Kaplansky introduces a property which he calls "hypothesis A". We now say that a valued field is a Kaplansky field if it satisfies hypothesis A. In [1], Kaplansky proves:

Theorem (Kaplansky (1942))

For a Kaplansky field, its maximal immediate extensions are unique up to valuation preserving isomorphism.

Generalized power series fields

To present a more precise version of this result, we need the definition of generalized power series fields.

Take an ordered abelian group G and a field $\mathbf{k}$. A factor set is a 2-cocycle $f : G \times G \to \mathbf{k}$, that is, it satisfies the following conditions:

(FS1) $f[\alpha,\beta] = f[\beta,\alpha]$,

(FS2) $f[0,0] = f[0,\alpha] = f[\alpha,0] = 1$,

(FS3) $f[\alpha,\beta+\gamma]f[\beta,\gamma] = f[\alpha+\beta,\gamma]f[\alpha,\beta]$,

(FS4) $f[-\alpha,\alpha] = 1$.

Generalized power series fields

The field $\mathbf{k}((G,f))$ of generalized power series with factor set f is the set of formal series

$$s = \sum_{\gamma \in G} a_\gamma t^\gamma$$

with $a_\gamma \in \mathbf{k}$, whose support

$$\operatorname{supp} s := \{\gamma \in G \mid a_\gamma \neq 0\}$$

is a well-ordered subset of G. Sums and products are defined formally, with the condition

$$t^\alpha t^\beta = f[\alpha,\beta] t^{\alpha+\beta}.$$

Generalized power series fields

A valuation on $\mathbf{k}((G,f))$ is given by

$$v(s) := \min \operatorname{supp} s$$

(by convention, $\min \operatorname{supp} s = \infty$ if $\operatorname{supp} s = \emptyset$). Its value group is G, and its residue field is $\mathbf{k}$. With this definition, $(\mathbf{k}((G,f)),v)$ is a maximal field.
If $f = 1$ we denote $\mathbf{k}((G,f))$ by $\mathbf{k}((G))$, or also by $\mathbf{k}((t^G))$.

In the paper [1], Kaplansky proves:

Theorem (Kaplansky (1942))

Every maximal Kaplansky field with value group G and residue field $\mathbf{k}$ is isomorphic to $\mathbf{k}((G,f))$ for a suitable factor set f.

Henselian and defectless valued fields

A valued field (K,v) is called henselian if for each algebraic extension $L|K$ the extension of v to L is unique. A finite extension $(L|K,v)$ of a henselian valued field (K,v) is called defectless if

$$[L:K] \;=\; (vL:vK)[Lv:Kv]\,.$$

A henselian field is called a defectless valued field if each of its finite extensions is defectless.

Every maximal valued field is henselian and defectless.

Henselizations

A henselization of (K, v) is an algebraic extension of (K, v) which admits a valuation preserving embedding in every other henselian extension of (K, v). Henselizations always exist and are unique up to valuation preserving isomorphism over K; therefore we talk of *the* henselization of (K, v) and denote it by $(K, v)^h = (K^h, v^h)$. The henselization of (K, v) is an immediate separable-algebraic extension. The valued field (K, v) is henselian if and only if it is equal to its henselization.

One more example of a valued field

Note that $\mathrm{char}\, K((t)) = \mathrm{char}\, K = \mathrm{char}\, Kv_t$. We call this the equal characteristic case. An important example of the complimentary mixed characteristic case is given by the field of p-adic numbers $\mathbb{Q}_p$ with its p-adic valuation v_p . Its residue field is $\mathbb{Q}_p v_p = \mathbb{F}_p$, so $\mathrm{char}\, \mathbb{Q}_p v_p = p$ while $\mathrm{char}\, \mathbb{Q}_p = 0$.

The valued fields $(\mathbb{F}_p((t)), v_t)$ and $(\mathbb{Q}_p, v_p)$ have much in common: both are maximal, hence henselian defectless, both have value group $\mathbb{Z}$, both have residue field $\mathbb{F}_p$. However, from a model theorist's view, $\mathbb{F}_p((t))$ is $\mathbb{Q}_p$'s naughty brother, as we will see soon.

A question about maximal immediate extensions

At the conference *Model-theoretic methods in non-archimedean geometry* in January 2025 in Münster, Nicolas Daans asked:

Question: Is every henselian defectless valued field $\mathcal{L}_{\mathrm{val}}$-existentially closed in its maximal immediate extensions?

Here, $\mathcal{L}_{\mathrm{val}}$ denotes the language of valued rings. The henselian field $\mathbb{F}_p(t)^h$ is known to be defectless and $\mathcal{L}_{\mathrm{val}}$-existentially closed in $\mathbb{F}_p((t))$, while it is an open problem whether the extension $\mathbb{F}_p((t))|\mathbb{F}_p(t)^h$ is $\mathcal{L}_{\mathrm{val}}$-elementary.

Questions about maximal immediate extensions

In the article
[2] K: *Elementary properties of power series fields over finite fields*, J. Symb. Logic **66** (2001), 771–791,
the following is shown:

Proposition (K (2001))

There are henselian defectless fields that are not $\mathcal{L}_{\mathrm{val}}$-existentially closed in their maximal immediate extensions.

Open problem 1: Is there a handy additional condition on the immediate extensions that remedies this situation?

Open problem 2: Is there a handy additional condition on maximal immediate extensions (L, v) of henselian defectless fields (K, v) that guarantees that $(K, v) \prec (L, v)$?

Note that if (K, v) is $\mathcal{L}_{\mathrm{val}}$-existentially closed in some maximal immediate extension, then it follows that (K, v) is henselian and defectless.

Properties of $\mathbb{F}_p((t))$

While model theoretic results about $\mathbb{Q}_p$ and in particular the decidability of $\mathbb{Q}_p$ are known since the work of Ax–Kochen and Ershov, we are still facing the

Open problem 3: What can we say about the model theory and in particular a complete axiomatization and the decidability of $(\mathbb{F}_p((t)), v_t)$?

In the article [2] the following negative result is proven:

Theorem (K (2001))

The recursive $\mathcal{L}_{\mathrm{val}}(t)$-elementary axiom system (A_t) "henselian defectless valued field of positive characteristic with value group a $\mathbb{Z}$-group (an ordered abelian group elementarily equivalent to $\mathbb{Z}$) with smallest element $v(t)$ and residue field $\mathbb{F}_p$" is not complete.

Properties of $\mathbb{F}_p((t))$

This theorem is proven by constructing an extension (L,v) of $(\mathbb{F}_p((t)),v_t)$ with the following properties:

- (L,v) satisfies axiom system (A_t) and $v(t)$ is the smallest positive element of vL,
- $L|K$ is of trancendence degree 1 and regular (i.e., $L|K$ is separable and K is relatively algebraically closed in L),
- there is an elementary $\mathcal{L}_{\mathrm{val}}(t)$-sentence which holds in (K,v) but not in (L,v).

Moreover, it is shown that (L,v) is not $\mathcal{L}_{\mathrm{val}}$-existentially closed in its maximal immediate extensions. This proves our above mentioned proposition.

Properties of $\mathbb{F}_p((t))$

In [2], a (not really handy) axiom scheme, called (PDOA), is suggested to be added to axiom system (A_t).
A much more elegant axiom scheme was found after Yuri Ershov introduced the notion of "extremal field" and claimed that $\mathbb{F}_p((t))$ is extremal. However, his definition and proof were faulty. In the article
[3] Salih Azgin – K – Florian Pop: *Characterization of extremal valued fields*, Proc. Amer. Math. Soc. **140** (2012), 1535–1547
it is shown that $\mathbb{F}_p((t))$ does not satisfy Ershov's definition, a corrected definition is given, and it is shown that $\mathbb{F}_p((t))$ satisfies this corrected definition, which we present now.

Extremal valued fields

A valued field (K, v) is called extremal if for every multi-variable polynomial $f(X_1, \ldots, X_n)$ over K, the set

$$\{v(f(a_1, \ldots, a_n)) \mid a_1, \ldots, a_n \in \mathcal{O}_K\} \subseteq vK \cup \{\infty\}$$

has a maximal element. This is an $\mathcal{L}_{\text{val}}$-elementary axiom scheme.

Theorem (Azgin – K – Pop (2012))

$\mathbb{F}_p((t))$ *is an extremal field.*

Open problem 4: Is (A_t) + "(K, v) is extremal" a complete axiom system?

Open problem 5: Is every extremal field existentially closed in its maximal immediate extensions?

Extremal valued fields

In the article

[4] Sylvy Anscombe - K: *Notes on extremal and tame valued fields*, J. Symb. Logic **81** (2016), 400–416,

an almost complete characterization of extremal valued fields is given:

Theorem (Anscombe – K (2016))

Let (K,v) be a nontrivially valued field. If (K,v) is extremal, then it is henselian and defectless, and

(i) *vK is a $\mathbb{Z}$-group, or*

(ii) *vK is divisible and Kv is large.*

Conversely, if (K,v) is henselian and defectless, and

(i) *$vK \simeq \mathbb{Z}$, or vK is a $\mathbb{Z}$-group and $\operatorname{char} Kv = 0$, or*

(ii) *vK is divisible and Kv is large and perfect,*

then (K,v) is extremal.

Extremal valued fields: more open problems

Open problem 6: Complete the characterization of extremal fields given in the theorem.

In contrast to properties such as "henselian" and "defectless", we do not entirely know how extremality behaves under composition of valuations:

Open problem 7: If $v = w \circ \bar{w}$ with w and $\bar{w}$ extremal and w has divisible value group, does it follow that v is extremal?

Open problem 8: We know that if $v = w \circ \bar{w}$ is extremal, then so is $\bar{w}$ (see Lemma 4.1 of [4]). But does it also follow that w is extremal?

It seems plausible to say that understanding the model theory of extremal fields is our best chance for understanding the model theory of $\mathbb{F}_p((t))$.

Decidability in mixed characteristic

Since Ax and Kochen, and independently Ershov, established in 1965 the decidability of the elementary theory of the field $\mathbb{Q}_p$ of p-adic numbers, several questions about the decidability of the elementary or the existential theory of local fields and their extensions have been answered, and several others have remained open, in particular in mixed characteristic, such as

- the totally ramified extension $\mathbb{Q}_p(\zeta_{p^\infty})$ obtained from $\mathbb{Q}_p$ by adjoining all p^n-th roots of unity, $n \in \mathbb{N}$,
- the totally ramified extension $\mathbb{Q}_p(p^{1/p^\infty})$ obtained from $\mathbb{Q}_p$ by adjoining a compatible system of p^n-th roots of p, $n \in \mathbb{N}$,
- the maximal abelian extension $\mathbb{Q}_p^{ab}$ of $\mathbb{Q}_p$.

These are studied in

[5] Konstantinos Kartas: *Decidability via the tilting correspondence*, Algebra and Number Theory **18** (2024), 209–248.

Decidability in mixed characteristic

Theorem (Kartas (2024))

The fields $\mathbb{Q}_p(\zeta_{p^\infty})$ *and* $\mathbb{Q}_p(p^{1/p^\infty})$ *equipped with their unique extensions of the* p*-adic valuation admit maximal immediate extensions which have decidable elementary* $\mathcal{L}_{\mathrm{val}}$*-theories.*

All of these maximal immediate extensions are tame fields. But the fields themselves are not even Kaplansky fields, and Kartas shows that there are uncountably many maximal immediate extensions with distinct elementary $\mathcal{L}_{\mathrm{val}}$-theories. This implies that uncountably many of them are not decidable.

Decidability in mixed characteristic

Open problem 9: What is the structure of these maximal immediate extensions? What are the indications in their structure that distinguish the decidable from the undecidable extensions?

Kartas also notes that $\mathbb{Q}_p^{ab}$ admits a unique maximal immediate extension, and that it follows from the model theory of algebraically maximal Kaplansky fields that this extension is $\mathcal{L}_{\mathrm{val}}$-decidable.

Truncation closed embeddings in power series fields

Let us discuss another incidence of "good" and "bad" maximal immedite extensions. It appears in the article

[6] K – Salma Kuhlmann – Antongiulio Fornasiero: *Towers of complements to valuation rings and truncation closed embeddings of valued fields*, J. of Algebra **323** (2010), 574–600.

There we study necessary and sufficient conditions for a valued field (K, v) with value group G and residue field $\mathbf{k}$ (with $\operatorname{char} K = \operatorname{char} \mathbf{k}$) to admit a truncation closed embedding in the field of generalized power series $\mathbf{k}((G,f))$ with factor set f. We define $\mathbf{k}(G,f)$ to be the subfield of $\mathbf{k}((G,f))$ generated by $\mathbf{k} \cup \{t^\gamma \mid \gamma \in G\}$.

Truncation closed embeddings

A subfield F of $\mathbf{k}((G,f))$ is **truncation closed** if whenever $s = \sum_{\gamma\in G} a_\gamma t^\gamma \in F$ and $g \in G$, the restriction

$$s_{<g} \; := \sum_{\gamma\in G^{<g}} a_\gamma t^\gamma$$

of s to the initial segment $G^{<g} := \{\gamma \in G \mid \gamma < g\}$ of G also belongs to F. Given a valued field (K,v) with value group G and residue field $\mathbf{k}$ with $\mathbf{k}(G,f) \subseteq K$, a truncation closed embedding of (K,v) in $\mathbf{k}((G,f))$ over $\mathbf{k}(G,f)$ is a valuation preserving embedding φ which is the identity on $\mathbf{k}(G,f)$ and such that $\varphi(K)$ is truncation closed.

Truncation closed embeddings

In the article [6] we prove:

Theorem (K – S. Kuhlmann – Fornasiero (2010))

Take a valued field (K, v) *with value group* G *and residue field* $\mathbf{k}$ *which admits a truncation closed embedding* φ *in* $\mathbf{k}((G, f))$ *for some factor set* f*. Then there is at least one maximal immediate extension and at least one maximal immediate algebraic extension of* (K, v) *that admits a truncation closed embedding in* $\mathbf{k}((G, f))$ *which extends* φ*.*

Truncation closed embeddings

Since the maximal immediate algebraic extensions of Kaplansky fields are unique up to (valuation preserving) isomorphism, we are able to derive:

Corollary

Every algebraically maximal Kaplansky field of positive characteristic with value group G and residue field $\mathbf{k}$ admits a truncation closed embedding in $\mathbf{k}((G,f))$ for some factor set f. In particular, every algebraically closed valued field of positive characteristic with value group G and residue field $\mathbf{k}$ admits a truncation closed embedding in $\mathbf{k}((G))$.

Good and bad maximal immediate extensions

On the other hand, we give an example of a truncation closed subfield of the power series field

$$\mathbf{k}((t^{\frac{1}{p^\infty}\mathbb{Z}}))\,,$$

where $\mathrm{char}\,\mathbf{k} > 0$, which has (at least) two different maximal immediate extensions, one of them admitting a truncation closed embedding extending the identity, and the other not. So here again we have "good" and "bad" maximal immediate extensions. The good one contains a root of $X^p - X - \frac{1}{t}$, while the bad one contains a root of

$$X^p - X - \left(\frac{1}{t} + y\right)$$

where y is an element of value 0 such that a root of $X^p - X - y$ extends the residue field.

Good and bad maximal immediate extensions

Open problem 10: Assume that (K,v) is a valued field with value group G and residue field $\mathbf{k}$ admitting a truncation closed embedding φ in $\mathbf{k}((G,f))$ for some factor set f. If (M,v) is a maximal immediate extension of (K,v) admitting a truncation closed embedding in $\mathbf{k}((G,f))$ extending φ, does it follow that $(K,v) \prec_{\exists} (M,v)$, and under which additional assumption does $(K,v) \prec (M,v)$ hold? Does the reverse implication hold?

Open problem 11: What does all this, and in particular the twist used in the construction of the mentioned example, tell us about the good and bad maximal immediate extensions that come up in Kartas' work? (Note that good and bad maximal immediate extensions also appear on the positive characteristic side of the perfectoid transfer theorem that Kartas uses.)

Back to our previous examples

Open problem 12: Are there also good and bad maximal immediate extensions of extremal fields?

While the extension (L, v) of $(\mathbb{F}_p((t)), v_t)$ we presented earlier has only bad maximal immediate extensions, it is itself constructed using twists somewhat similar to the one we just mentioned.

Open problem 13: Knowing that $(\mathbb{F}_p((t)), v_t)$ is an extremal field, what does the construction of (L, v) tell us about the model theory of extremal fields?

Tame and roughly tame fields

A henselian field (K, v) is a tame field if and only if the following conditions hold:

(TF1) if char $Kv = p > 0$, then vK is p-divisible,

(TF2) Kv is perfect,

(TF3) (K, v) is algebraically maximal.

Replacing (TF1) by

(TF1r) if char $Kv = p > 0$, then $[-v(p), v(p)]$ is p-divisible,

we obtain the definition of a roughly tame field.

Henselian Rationality

A function field (F, v) over (K, v) of transcendence degree 1 is henselian rational if there is some $x \in F$ such that $F \subset K(x)^h$.
In the article

[7] K: *Elimination of Ramification II: Henselian Rationality*, Israel J. Math. **234** (2019), 927–958,

the following is proven:

Theorem

Let (K, v) be a tame field and $(F|K, v)$ an immediate function field. If its transcendence degree is 1, then $(F|K, v)$ is henselian rational.

Henselian Rationality

In [2], immediate function fields of transcendence degree 1 over (L, v) are constructed that are not henselian rational. While (L, v) is henselian defectless, L is not perfect. So there is still hope for a positive solution of the following

Open problem 14: Does the Henselian Rationality theorem also hold over perfect fields of positive characteristic?

If it holds, it would be a key to an alternative proof of Temkin's "Inseparable Local Uniformization".

Roughly tame fields and the defect

In the article

[8] Anna Rzepka – Piotr Szewczyk: *Defect extensions and a characterization of tame fields*, J. Algebra **630** (2023), 68–91,

the following is proven:

Theorem (Rzepka – Szewczyk (2023))

A henselian field is roughly tame if and only if all of its algebraic extension fields are defectless fields.

Roughly deeply ramified fields

We would like to generalize the last theorem to a larger class of valued fields. We call (K, v) a roughly deeply ramified field, or in short an rdr field, if it satisfies the following axioms:

(DRvr) if char $Kv = p > 0$, then

- the Frobenius $x \mapsto x^p$ is surjective on $\mathcal{O}_K / p\mathcal{O}_K$ if char $K = 0$,
- the completion $\widehat{K}$ of K w.r.t. v is perfect if char $K = p$,

(DRvp) if char $Kv = p > 0$, then $v(p)$ is not the smallest positive element in the value group vK.

The two axioms (DRvp) and (DRvr) together imply that the smallest convex subgroup of vK containing $v(p)$ (or equivalently, the interval $[-v(p), v(p)]$) is p-divisible.

Every perfect valued field of positive characteristic is roughly deeply ramified.

Roughly deeply ramified fields

Roughly deeply ramified fields are in general not defectless. However, their Galois extensions $\mathcal{E} = (L|K,v)$ of prime degree p only admit a specific type of defect. Assume that $\mathcal{E}$ has nontrivial defect. For every σ in its Galois group $\mathrm{Gal}\,(L|K)$, with $\sigma \neq \mathrm{id}$, we set

$$\Sigma_\sigma \;:=\; \left\{ v\left(\frac{\sigma b - b}{b}\right) \,\middle|\, b \in L^\times \right\} .$$

This set is a final segment of vK and independent of the choice of σ; we denote it by $\Sigma_{\mathcal{E}}$. We say that $\mathcal{E}$ has independent defect if $\Sigma_{\mathcal{E}}$ is equal to $\{\alpha \in vK \mid \alpha > H_{\mathcal{E}}\}$ for some proper convex subgroup $H_{\mathcal{E}}$ of vK such that $vK/H_{\mathcal{E}}$ has no smallest positive element; otherwise we say that $\mathcal{E}$ has dependent defect.

Roughly deeply ramified fields

For background on roughly deeply ramified fields, see

[9] K – Anna Rzepka: *The valuation theory of deeply ramified fields and its connection with defect extensions*, Transactions Amer. Math. Soc. **376** (2023), 2693–2738.

There, the following is proven:

Theorem

Take a roughly deeply ramified field (K, v) *of residue characteristic* $p > 0$. *If* $\operatorname{char} K = 0$, *then assume that* K *contains a primitive* p*-th root of unity. Then for all algebraic extensions* (L, v) *of* (K, v), *all Galois extensions of* (L, v) *of degree* p *with nontrivial defect have independent defect.*

Open problem 15: Is the converse also true?

Rational place = existentially closed?

In the article

[10] K: *On places of algebraic function fields in arbitrary characteristic*, Advances in Math. **188** (2004), 399–424,

the following question is studied: Take a field extension $F|K$ such that F admits a K-rational place, or in other words, a valuation with residue field K. Under which additional conditions does it follow that K is existentially closed in F?

Here a key role is played by large fields. While they are usually defined in a different way, one can also use the model theoretic approach: A field K is large if it is existentially closed in $K((t))$.

Rational place = existentially closed?

Theorem (K (2004))

Let K be a perfect field. Then the following conditions are equivalent:
1) K is a large field,
2) K is existentially closed in every power series field $K((t^G))$,
3) K is existentially closed in every extension field L which admits a K-rational place.

Local uniformization is a local form of resolution of singularities.

Theorem (K (2004))

If all rational places of arbitrary function fields admit local uniformization, then the three conditions of the previous theorem are equivalent, for arbitrary fields K.

A conditional result about $\mathbb{F}_p((t))$

In the paper
[11] Sylvy Anscombe – Philip Dittmann – Arno Fehm: *Axiomatizing the existential theory of $F_q((t))$*, Algebra and Number Theory **17** (2023), 2013–2032,

the assumption that implication 1)⇒3) holds for *arbitrary* fields K is called hypothesis (R4). Hence local uniformization implies (R4). The authors prove:

Theorem (Anscombe – Dittmann – Fehm (2023))

If (R4) holds, then the existential $\mathcal{L}_{\mathrm{val}}(t)$-theory of $\mathbb{F}_p((t))$ is decidable.

By now, many other results have been shown with the help of (R4).

Open problem 16: Does (R4) hold?

Alteration

De Jong has proved resolution by alteration. Alteration means that a finite extension of the function field of the algebraic variety under consideration is taken into the bargain. By valuation theoretical tools, Hagen Knaf and K have proved local uniformization by alteration.

Open problem 17: Does local uniformization by alteration imply a reasonable (and useful) hypothesis "(R4) by alteration"? What could be a "model theory by alteration"?

You cannot always get what you want

Alteration

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You cannot always get what you want – but perhaps after a finite extension?

A first answer

In the manuscript

[12] Hagen Knaf – K: *Large imperfect fields are existentially closed in function fields after finite constant extension*, in preparation,

the following result is proven:

Theorem

Take a large field K and a function field $F|K$ which admits a rational place. Then there is a finite purely inseparable extension $K'|K$ such that every purely inseparable extension K'' of K' is existentially closed in $F.K''$.

That's it

THE END

Thank you for your attention!

More detailed information

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https://www.fvkuhlmann.de/Fvkslides.html

and a lecture series on valued function fields and the defect can be found on the web page

https://www.fvkuhlmann.de/Fvkls.html.

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