The theory of the defect and its application to the problem of local uniformization, IV

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Padova, March 12, 2021

The Fundamental Inequality

To start with, we will reconsider the definition of "defectless field". To this end, we introduce the general form of the Fundamental Inequality.

Take a valued field (K, v) and a finite extension $L|K$. Recall that the set of all extensions of v from K to L is

$$\{\tilde{v}\sigma \mid \sigma \text{ an embedding of } L \text{ in } \tilde{K} \text{ over } K\} .$$

Therefore, as $L|K$ is finite, the number g of distinct extensions of v from K to L is smaller or equal to the extension degree $[L : K]$. More precisely, g is smaller or equal to the degree of the maximal separable subextension of $L|K$. In particular, if $L|K$ is purely inseparable, then v has a unique extension from K to L.

The Fundamental Inequality

We will denote the distinct extensions of v from K to L by $v_1, \ldots, v_{\mathrm{g}}$. For each $i \in \{1, \ldots, \mathrm{g}\}$, $\mathrm{e}_i := (v_i L : vK)$ is the ramification index and $\mathrm{f}_i := [Lv_i : Kv]$ is the inertia degree of $(L|K, v_i)$.

Theorem (Fundamental Inequality)

Assume that $n := [L : K]$ is finite. Then for every $i \in \{1, \ldots, \mathrm{g}\}$, e_i and f_i are finite, and the following holds:

$$n \geq \sum_{i=1}^{\mathrm{g}} \mathrm{e}_i \mathrm{f}_i \,. \tag{1}$$

Defectless extensions revisited

We will say that L is defectless over (K, v) if equality holds in the Fundamental Inequality (1). The following transitivity holds:

Lemma

Take any valued field (K, v) and finite extensions $L|K$ and $N|L$. Then N is defectless over (K, v) if and only if L is defectless over (K, v) and for every extension v_i of v from K to L, N is defectless over (L, v_i).

Defectless fields revisited

An arbitrary valued field (K, v) is called a defectless field if every finite extension L of K is defectless over (K, v). It is called a separably defectless field if this holds for every finite separable extension, and an inseparably defectless field if this holds for every finite purely inseparable extension.

The following theorem shows that this definition for "defectless" is compatible with our earlier one.

Theorem

Take a valued field (K, v) and a henselization (K^h, v). Then (K, v) is a defectless field if and only if (K^h, v) is. The same holds for "separably defectless" and for "inseparably defectless" in place of "defectless".

In this lecture, we will discuss the following theorem and sketch parts of its proof.

Theorem

(Generalized Stability Theorem)
Assume that v is an Abhyankar valuation on the function field $F|K$. If (K,v) is a defectless field, then (F,v) is a defectless field. The same holds for "inseparably defectless" in place of "defectless". If vK is cofinal in vF, then it also holds for "separably defectless" in place of "defectless".

From now on, we will abreviate "Generalized Stability Theorem" by "GST".

The Generalized Stability Theorem

As a consequence of the GST, we find that even if nothing is assumed about (K, v), defects in finite extensions of (F, v) can be "cancelled out" by suitable finite extensions of K.

Corollary

Assume that v is an Abhyankar valuation on the function field $F|K$, and $E|F$ is a finite extension. Fix an extension of v from F to $\tilde{K}.F$. Then there is a finite extension $L_0|K$ such that for every algebraic extension L of K containing L_0, $L.E$ is defectless over $(L.F, v)$. If (K, v) is henselian, then $L_0|K$ can be chosen to be purely wild, i.e., linearly disjoint from $K^r|K$.

Here, the compositum $L.F$ of two subfields L and F of a common field extension (such as $\tilde{F}$ in the present case) is the smallest subfield of this common extension that contains both L and F.

History of the Generalized Stability Theorem

An early forerunner of the GST was proved by H. Grauert and R. Remmert and was restricted to the case of algebraically closed complete ground fields of rank 1. A generalization of the Grauert–Remmert Stability Theorem was given by L. Gruson. A good presentation of it can be found in the book *Non-Archimedean Analysis* of S. Bosch, U. Güntzer and R. Remmert (§5.3.2, Theorem 1). The proof uses methods of non-archimedean analysis. Further generalizations are due to M. Matignon and J. Ohm.

History of the Generalized Stability Theorem

Ohm arrived at a quite general version of the Stability Theorem. But like all of its forerunners, it is still restricted to the case of $\mathrm{trdeg}(F|K) = \mathrm{trdeg}(Fv|Kv)$ (the case of constant reduction) and is therefore not sufficient for the applications we will list below. Ohm deduces his theorem from Proposition 3 on page 215 of the book of Bosch, Güntzer and Remmert (more precisely, from a generalized version of this proposition which is proved but not stated in the book).

The name "Stability Theorem" originates from the fact that in the tradition of non-archimedean analysis, defectless fields are called "stable fields".

Method of proof

In contrast, we give a proof of the GST which replaces the methods from non-archimedean analysis used by the forerunners by valuation theoretical arguments. Such arguments seem to be more adequate for a theorem that is of valuation theoretical nature. First, using ramification theory we reduce the proof to the study of Galois extensions of degree p of special henselized function fields F. Then we deduce normal forms which allow us to read off that the extensions are defectless.

Our approach has some similarity with Abhyankar's method of using ramification theory in order to reduce resolution of singularities to the study of Galois extensions of degree p, and his search for suitable normal forms of Artin–Schreier–like minimal polynomials.

Application: Elimination of Ramification

The GST is used to provide a solution to our problem of Elimination of Ramification in the case of Abhyankar valuations.

Take a function field $F|K$ and an Abhyankar valuation v of $F|K$ such that (K, v) is a defectless field. Assume that $Fv|Kv$ is a separable extension and vF/vK is torsion free.

Application: Elimination of Ramification

Theorem

Under the above assumptions, $(F|K,v)$ admits elimination of ramification in the following sense: there is a transcendence basis

$$\mathcal{T} = \{x_1,\ldots,x_\rho,y_1,\ldots,y_\tau\} \tag{2}$$

of $(F|K,v)$ such that

$$vF = vK(\mathcal{T}) = vK \oplus \mathbb{Z}vx_1 \oplus \ldots \oplus \mathbb{Z}vx_\rho \tag{3}$$

and

$$y_1v,\ldots,y_\tau v \text{ form a separating transcendence basis of } Fv|Kv\,, \tag{4}$$

and for each such transcendence basis $\mathcal{T}$ and every extension of v to the algebraic closure of F, (F,v) lies in the absolute inertia field of $(K(\mathcal{T}),v)$. In other words, $(F|K,v)$ is inertially generated.

This theorem in turn is a crucial ingredient in the proof of the following theorem:

Theorem (Knaf&K, 2005)

Take a function field $F|K$ and an Abhyankar place P on $F|K$. If $FP|K$ is separable, then $(F|K,P)$ admits Local Uniformization.

Another application: an embedding theorem

In model theory algebra, a good way to prove results is to base them on the algebraic structure theory of the algebraic objects under consideration. For the model theory of valued fields, an efficient way is to use embedding theorems based on the structure theory of valued function fields. The Elimination of Ramification for Abhyankar valuations can be used to prove the following embedding theorem:

Another application: an embedding theorem

Theorem

Take a function field $F|K$ and an Abhyankar valuation v of $F|K$ such that (K,v) is a defectless field. Assume that vF/vK is torsion free and $Fv|Kv$ is a separable extension. Assume further that (K^*,v) is a henselian extension of (K,v) and there are embeddings

$$\iota_v : \; vF \hookrightarrow vK^* \quad \text{fixing the elements of } vK$$

and

$$\iota_r : \; Fv \hookrightarrow K^*v \quad \text{fixing the elements of } Kv\,.$$

Then these embeddings can be lifted to a valuation preserving embedding of (F,v) in (K^*,v) which fixes the elements of K.

We will give a quick sketch of the proof.

Bourbaki helps

We recall the following theorem from the last lecture. Let $(L|K,v)$ be an extension of valued fields. Take elements $x_i, y_j \in L$, $i \in I$, $j \in J$, such that the values vx_i, $i \in I$, are rationally independent over vK, and the residues $y_j v$, $j \in J$, are algebraically independent over Kv. If we write

$$f = \sum_k c_k \prod_{i\in I} x_i^{\mu_{k,i}} \prod_{j\in J} y_j^{\nu_{k,j}} \in K[x_i, y_j \mid i \in I, j \in J]$$

in such a way that for every $k \neq \ell$ there is some i such that $\mu_{k,i} \neq \mu_{\ell,i}$ or some j such that $\nu_{k,j} \neq \nu_{\ell,j}$, then

$$vf = \min_k v\, c_k \prod_{i\in I} x_i^{\mu_{k,i}} \prod_{j\in J} y_j^{\nu_{k,j}} = \min_k vc_k + \sum_{i\in I} \mu_{k,i} vx_i \,. \qquad (5)$$

Bourbaki helps

In particular,

$$\begin{aligned} vK(x_i, y_j \mid i \in I, j \in J) &= vK \oplus \bigoplus_{i \in I} \mathbb{Z} vx_i\,, \\ K(x_i, y_j \mid i \in I, j \in J)v &= Kv\,(y_j v \mid j \in J)\,, \end{aligned}$$

and the valuation v on $K(x_i, y_j \mid i \in I, j \in J)$ is uniquely determined by its restriction to K, the values vx_i and the residues $y_j v$.

Sketch of the proof for the embedding theorem

Now assume that v is an Abhyankar valuation on the function field $F|K$ such that (K,v) is a defectless field. Take a transcendence basis

$$\mathcal{T} \;=\; \{x_1,\ldots,x_\rho,y_1,\ldots,y_\tau\}$$

satisfying conditions (3) and (4) as in the Elimination of Ramification theorem. Choose elements $x_i^* \in K^*$ such that

$$vx_i^* \;=\; \iota_v(vx_i)\;,$$

and $y_j^* \in K^*$ such that

$$y_j^*v \;=\; \iota_r(y_jv)\;.$$

Then by the Bourbaki theorem we just reviewed, sending x_i to x_i^* and y_j to y_j^* induces a valuation preserving embedding ι_0 of $(K(\mathcal{T}),v)$ in (K^*,v).

Sketch of the proof for the embedding theorem

By our choice of $\mathcal{T}$, the finite extension $Fv|K(\mathcal{T})v$ is separable. By the Theorem of the Primitive Element, we can pick some $\zeta \in Fv$ such that $Fv = (K(\mathcal{T})v)(\zeta)$. Then we choose a monic polynomial f with coefficients in $K(\mathcal{T})$ whose reduction under v is the minimal polynomial of ζ over $K(\mathcal{T})v$. Using Hensel's Lemma, we find a root $z \in F^h$ such that $zv = \zeta$. We take f^* to be the polynomial f with its coefficients replaced by their images under ι_0. Then f^*v is the minimal polynomial of $\iota_r(\zeta)$ over the residue field of $\iota_0(K(\mathcal{T}))$. As (K^*, v) is assumed to be henselian, Hensel's Lemma can be used to find a root z^* of f^* with $z^*v = \iota_r(\zeta)$.

Sketch of the proof for the embedding theorem

Now we can extend ι_0 to an embedding ι_1 of $K(\mathcal{T},z)$ by sending z to z^*. Since

$$[K(\mathcal{T},z) : K(\mathcal{T})] \;=\; [(K(\mathcal{T})v)(\zeta) : K(\mathcal{T})v]\,,$$

the Fundamental Inequality shows that the extension of v from $K(\mathcal{T})$ to $K(\mathcal{T},z)$ is unique. This implies that ι_1 is valuation preserving.

By the universal property of the henselization the embedding ι_1 of $K(\mathcal{T},z)$ can be extended to an embedding ι of $K(\mathcal{T},z)^h$ in the henselian field (K^*,v).

Sketch of the proof for the embedding theorem

By our choice of $\mathcal{T}$ and the element z, the extension $(F^h|K(\mathcal{T},z)^h,v)$ is immediate. On the other hand, the GST shows that $(K(\mathcal{T},z),v)$ is a defectless field, and the same holds for $(K(\mathcal{T},z)^h,v)$. Therefore, $F^h = K(\mathcal{T},z)^h$. Now the restriction of ι to F is the embedding we are looking for.

To show how this embedding theorem leads to a model theoretic result, we present the following Ax–Kochen–Ershov Principle that is derived from it:

Theorem

Take a henselian defectless field (K, v) and a valued field extension $(L|K, v)$ without transcendence defect. If vK is existentially closed in vL (in the language of ordered groups) and Kv is existentially closed in Lv (in the language of fields), then (K, v) is existentially closed in (L, v) (in the language of valued fields).

For (K, v) to be existentially closed in (L, v) it suffices that (K, v) is existentially closed in every finitely generated subextension. Such subextensions are valued function fields. Our embedding theorem provides embeddings of them in certain elementary extensions (K^*, v) of (K, v).

Proof of the GST: some basics about the defect

Here are some basic facts we use in several instances in the proof of the GST. Recall that the defect is multiplicative: if $(L|K,v)$ and $(N|L,v)$ are finite unibranched extensions, then

$$\mathrm{d}(N|K,v) \;=\; \mathrm{d}(N|L,v)\cdot \mathrm{d}(L|K,v)\,.$$

Further:

Lemma

If $L|K$ is a finite extension and (K,v) is a defectless field, then also (L,v) is a defectless field.

This follows from the transitivity of the property "defectless extension" that we have already mentioned. In order to prove the first assertion of the GST, it thus suffices to prove it for the rational function field generated by any transcendence basis of the function field.

Another tool: valuation disjoint extensions

We now present a tool that serves in two reduction steps in the proof of the GST.

Recall that two subextensions $L|K$ and $F|K$ of some "universal" field extension $\Omega|K$ are linearly disjoint if any number of elements in F that are linearly independent over K remain linearly independent over L. Passing to a suitable notion of valuation independence of elements, one analogously defines the notion of valuation disjoint subextensions $(L|K, v)$ and $(F|K, v)$ of some fixed extension $(\Omega|K, v)$.

Characterization of valuation disjoint extensions

Lemma

Let $(\Omega|K, v)$ be an extension of valued fields and $F|K$ and $L|K$ be subextensions of $\Omega|K$. Then $(F|K, v)$ and $(L|K, v)$ are valuation disjoint if and only if

1) $vF|vK$ and $vL|vK$ are disjoint in $v\Omega$, and

2) $Fv|Kv$ and $Lv|Kv$ are linearly disjoint in Ωv.

The first condition means that any number of elements in vF that lie in distinct cosets modulo vK also lie in distinct cosets modulo vL.

Here is the reason why we consider valuation disjoint extensions:

Proposition

Take an extension $(F|K,v)$ and a finite unibranched extension $(L|K,v)$ that are valuation disjoint in $(\tilde{F},v)$. If also $(L.F|F,v)$ is unibranched, then

$$\mathrm{d}(L.F|F,v) \;\leq\; \mathrm{d}(L|K,v)\,. \tag{6}$$

Valuation regular extensions

Recall that an extension $F|K$ is called regular if $F|K$ and $\tilde{K}|K$ are linearly disjoint in $\tilde{F}|K$, or equivalently, if $F|K$ is separable and K is relatively algebraically closed in F. We say that an extension $(F|K,v)$ is valuation regular if $(F|K,v)$ and $(\tilde{K}|K,v)$ are valuation disjoint in $(\tilde{F}|K,v)$ (this does not depend on the extension of v from F to $\tilde{F}$), or equivalently, if

1) vF/vK is torsion free, and

2) $Fv|Kv$ is regular.

Every valued field extension of an algebraically closed valued field is valuation regular.

An important example

Take a function field $F|K$ with an Abhyankar valuation v. As before, choose a transcendence basis

$$\mathcal{T} = \{x_1, \ldots, x_\rho, y_1, \ldots, y_\tau\}$$

such that
the values $vx_1, \ldots, vx_\rho$ are rationally independent over vK, and
the residues $y_1v, \ldots, y_\tau v$ are algebraically independent over Kv.

Then by the Bourbaki theorem,

$$\begin{aligned} vK(\mathcal{T}) &= vK \oplus \bigoplus_{1\leq i\leq \rho} \mathbb{Z}vx_i\,, \\ K(\mathcal{T})v &= Kv\,(y_jv \mid 1 \leq j \leq \tau)\,. \end{aligned}$$

It follows that $vK(\mathcal{T})/vK$ is torsion free and $K(\mathcal{T})v|Kv$ is regular. This shows that the extension $(K(\mathcal{T})|K, v)$ is valuation regular.

Valuation regular extensions

Proposition

Take a valuation regular extension $(F|K,v)$ and fix an extension of v from F to $\tilde{F}$. Then the following assertions hold:

1) If (K,v) and $(\tilde{K}.F,v)$ are defectless fields, then also (F,v) is a defectless field.

2) If (K,v) and $(K^{1/p^\infty}.F,v)$ are inseparably defectless fields, then also (F,v) is an inseparably defectless field.

Here K^{1/p^∞} denotes the perfect hull of K.

Proof of the GST: reduction steps

The proof of the GST involves several reduction steps:
1) Reduction to algebraically closed ground fields.
2) Reduction to transcendence degree 1.
3) Reduction to finite rank.
4) Reduction to rank 1.
5) Reduction to Galois extensions of degree $p = \mathrm{char}\, Kv$.

We will now sketch each of the reduction steps, as well as some tools for the proof, and some special cases.

Reduction to algebraically closed ground fields

For our function field $F|K$ with Abhyankar valuation v, we choose the transcendence basis $\mathcal{T}$ that we have just shown to generate a valuation regular extension $(K(\mathcal{T})|K,v)$. As (K,v) is assumed to be a defectless field, part 1) of our theorem on valuation regular extensions shows that in order to prove the first assertion of the GST it suffices to show that $(\tilde{K}(\mathcal{T}),v)$ is a defectless field.

The "inseparably defectless" case

Similarly, part 2) of our theorem on valuation regular extensions shows that in order to prove that $(K(\mathcal{T}), v)$ is an inseparably defectess field it suffices to show that $(K^{1/p^\infty}(\mathcal{T}), v)$ is an inseparably defectless field. However, how can we then conclude that the original function field (F, v) is an inseparably defectess field? After all, in this case we cannot use the transitivity as we have done before, since we could only deal with purely inseparable extensions, while $F|K(\mathcal{T})$ may not be purely inseparable.

The "inseparably defectless" case

Let us discuss this problem right now, so that we can get inseparable extensions out of the way once and for all. We ask: if (E, v) is an inseparably defectless field, does the same hold for every finite extension (F, v)? (We assume that $\mathrm{char}\, E = p > 0$ since otherwise everything is trivial.) The answer is *yes*, and there is an easy proof if

$$[E : E^p]$$

is finite, in which case we say that E has finite p-degree. Recall that in characteristic $p > 0$, the function $a \mapsto a^p$ is a field homomorphism, called the Frobenius. Therefore, E^p is indeed a subfield of E.

The "inseparably defectless" case

The Frobenius also sends the extension $E^{1/p}|E$ onto the extension $E|E^p$, and likewise for every $i \in \mathbb{N}$, the extension $E^{1/p^{i+1}}|E^{1/p^i}$ onto the extension $E^{1/p^i}|E^{1/p^{i-1}}$. Consequently, if $(E|E^p, v)$ is defectless, then so is every extension $(E^{1/p^{i+1}}|E^{1/p^i}, v)$. If E has finite p-degree, then each of these unibranched extensions is finite. By the multiplicativity of the defect, we conclude: if $(E|E^p, v)$ is defectless, then also $(E^{1/p^i}|E, v)$ is defecless for every $i \in \mathbb{N}$; as every purely inseparable extension of E can be embedded in E^{1/p^i} for large enough i, this implies that (E, v) is an inseparably defectless field.

The “inseparably defectless” case

If finite, then the unibranched extension $(E|E^p, v)$ is defectless if and only if

$$[E : E^p] = (vE : pvE)[Ev : (Ev)^p] \tag{7}$$

since $vE^p = pvE$ and $E^pv = (Ev)^p$. This proves:

Theorem

A valued field (E, v) of characteristic p and finite p-degree is an inseparably defectless field if and only if it satisfies equation (7).

The "inseparably defectless" case

If $F|E$ is a finite extension, then $[F:F^p] = [E:E^p]$, $(vF : pvF) = (vE : pvE)$, and $[Fv : (Fv)^p] = [Ev : (Ev)^p]$. This shows:

Corollary

Take a finite extension $(F|E, v)$ of valued fields of characteristic $p > 0$. If (E, v) is an inseparably defectless field of finite p-degree, then so is (F, v), and vice versa.

Hence in order to prove the "inseparably defectless" case it suffices to prove that if K is perfect, then $(K(\mathcal{T}), v)$ is an inseparably defectless field of finite p-degree.

The "inseparably defectless" case

Since K is perfect and $\mathcal{T}$ contains $\rho + \tau$ many elements that are algebraically independent over K,

$$[K(\mathcal{T}) : K(\mathcal{T})^p] \;=\; [K(\mathcal{T}) : K(\mathcal{T}^p)] \;=\; p^{\rho+\tau}\,.$$

Hence $K(\mathcal{T})$ has finite p-degree. Likewise, as vK is p-divisible and the values vx_i are rationally independent modulo vK,

$$(vK(\mathcal{T}) : pvK(\mathcal{T})) \;=\; (vK \oplus \bigoplus_{1\leq i\leq\rho} \mathbb{Z}vx_i \;:\; vK \oplus \bigoplus_{1\leq i\leq\rho} p\mathbb{Z}vx_i) \;=\; p^\rho\,,$$

and as Kv is perfect and the residues y_jv are algebraically independent over Kv,

$$[K(\mathcal{T})v : (K(\mathcal{T})v)^p] = [Kv(y_1v,\ldots,y_\tau v) : Kv((y_1v)^p,\ldots,(y_\tau v)^p)] = p^\tau.$$

This shows that $(K(\mathcal{T}), v)$ satisfies equation (7), proving that it is an inseparably defectless field.

Reduction to transcendence degree 1

The reduction to transcendence degree 1 proceeds by induction on the number $\rho + \tau$ in $\mathcal{T}$. We are left to deal with two different cases:

1) a value-transcendental extension $(K(t)|K, v)$ with vt non-torsion over vK,

2) a residue-transcendental extension $(K(t)|K, v)$ with tv transcendental over Kv.

In order to show that $(K(t), v)$ is a defectless field, it suffices to show that $(K(t)^h, v)$ is a defectless field.

Reduction to finite rank

If $(K(t)^h(a_1,\ldots,a_k)|K(t)^h,v)$ is any finite extension, then we have to show that it is defectless. To say that the elements $a_1,\ldots,a_k$ are algebraic over $K(t)^h$, it suffices to say that they are algebraic over $K(t)$. To do so, we only need finitely many elements from $K(t)$ as coefficients for the minimal polynomials of the a_i over $K(t)$. These finitely many rational functions from $K(t)$ in turn need finitely many coefficients from K. These finitely many elements from K give rise to a finitely generated extension k_0 of the prime field $\mathbb{F}_p$ of K. We take k to be the algebraic closure of k_0 (which is contained in the algebraically closed field K). We obtain that also $(k(t)^h(a_1,\ldots,a_k)|k(t)^h,v)$ is a finite extension.

Reduction to finite rank

The finitely generated extension $k_0|\mathbb{F}_p$ has finite transcendence degree, and the same holds for $k(t)^h|\mathbb{F}_p$. As v is trivial on $\mathbb{F}_p$ (or has rank at most 1 if $p = 0$, in which case $\mathbb{F}_p = \mathbb{Q}$), it follows that the rank of $(k(t)^h, v)$ must be finite.

Reduction to finite rank

One can prove that the extensions $(k(t)^h(a_1,\ldots,a_k)|k(t)^h,v)$ and $(K(t)^h|k(t)^h,v)$ are valuation disjoint in $(K(t)^h(a_1,\ldots,a_k)|k(t)^h,v)$. Now the proposition on the behaviour of the defect under valuation disjoint extensions shows that

$$\mathrm{d}(K(t)^h(a_1,\ldots,a_k)|K(t)^h,v) \;\leq\; \mathrm{d}(k(t)^h(a_1,\ldots,a_k)|k(t)^h,v)\,.$$

Hence if we have already shown that the valued field $(k(t)^h,v)$ of finite rank is a defectless field, then the same follows for $(K(t)^h,v)$.

References

- Bosch, S. – Güntzer, U. – Remmert, R.: *Non-Archimedean Analysis*, Berlin (1984)
- Grauert, H. – Remmert, R.: *Über die Methode der diskret bewerteten Ringe in der nicht archimedischen Analysis*, Invent. Math. **2** (1966), 87–133
- Gruson, L.: *Fibrés vectoriels sur un polydisque ultramétrique*, Ann. Sci. Ec. Super., IV. Ser., **177** (1968), 45–89

References

- Knaf, H. — Kuhlmann, F.-V.: *Abhyankar places admit local uniformization in any characteristic*, Ann. Scient. Ec. Norm. Sup., **38** (2005), 833–846
- Kuhlmann, F.-V.: *Elimination of Ramification I: The Generalized Stability Theorem*, Trans. Amer. Math. **362** (2010), 5697–5727
- Ohm, J.: *The henselian defect for valued function fields*, Proc. Amer. Math. Soc. **107** (1989), 299–308