Sums of squares in semidefinite optimization

Semidefinite programming is a highly efficient tool for optimizing linear functions over suitable semi-algebraic domains $S \subseteq \mathbb{R}^n$. The moment relaxation method, introduced by Lasserre and Parrilo about ten years ago, shows that the optimization becomes exact whenever the linear polynomials non-negative on S have weighted sum of squares representations of uniformly bounded degrees. We show that this latter condition, suitably generalized, is also necessary for the optimization to become exact. As a consequence we disprove the Helton-Nie conjecture, which means that there exist convex domains K that do not have a semidefinite representation. The smallest examples of such K that we know of have dimension 14.

Claus Scheiderer
Univ Konstanz (Germany)
claus.scheiderer@uni-konstanz.de