Resolution of Singularities of Arithmetic Threefolds

Vincent COSSART (U. Versailles LMV UMR8100)

In a joint work with Olivier Piltant (CNRS, LMV-UMR 8100), we have proved the following Resolution of Singularities Theorem for arithmetical varieties of dimension three:

Theorem. *Let C be an excellent curve and $\mathcal{X}$ be a reduced projective scheme over C, all whose local rings have dimension at most three. There exists a projective birational C-morphism $\pi : \mathcal{X}' \to \mathcal{X}$ with the following properties:*

(1) $\mathcal{X}'$ *is everywhere regular;*

(2) π *induces an isomorphism* $\pi^{-1}(\mathrm{Reg}(\mathcal{X})) \simeq \mathrm{Reg}(\mathcal{X}')$*;*

(3) $\pi^{-1}(\mathrm{Sing}(\mathcal{X}))$ *is a normal crossings divisor on* $\mathcal{X}'$*.*

I will focus on one essential point: the definition of the main differential invariant.

The initial forms of the equation of the singularity with respect to some special monomial valuations are well defined and give rise to our invariant. Despite all the invariance properties of these initial forms, there is some freedom in the choice of well adapted coordinates which may change, in some very special case, the possible values of the invariant. We will discuss a very bizarre example.