On the fibres of varieties over valuation domains

Hagen Knaf

I will discuss various properties of the fibres of a morphism $f : X \to \mathrm{Spec}(R)$ of finite presentation, where X is an integral scheme and R is a valuation domain. Among them are: dimension, connectedness, reducedness, singularities. The investigation is driven by the wish to understand the structure of regular schemes X of finite presentation over a valuation domain R – of course "modulo the noetherian case". A result in that direction will be presented in the talk. Here the possibly non-noetherian scheme X is called regular if for every $x \in X$ and every finitely generated ideal $I \subseteq O_{X,x}$ the projective dimension $\mathrm{pdim}_O(I)$ is finite.

The work is inspired by the results on normal curves over valuation domains and their relation to the valuation theory of function fields obtained by B.Green, M.Matignon and F.Pop in a sequence of articles.

References

[GMP1] B.W.Green, M.Matignon, F.Pop, *On valued function fields I*, Manuscr. Math. **65**(1989), 257-276.

[GMP2] B.W.Green, M.Matignon, F.Pop, *On valued function fields II, Regular functions and elements with the uniqueness property*, J. Reine Angew. Math. **412**(1990), 128-149.

[GMP3] B.W.Green, M.Matignon, F.Pop, *On valued function fields III, Reductions of algebraic curves*, J. Reine Angew. Math. **432**(1992), 117-133.

[GMP4] B.W.Green, M.Matignon, F.Pop, *On the Local Skolem Property*, J. Reine Angew. Math. **458**(1995), 183-199.

[G1] B.Green, *The Relative Genus Inequality for Curves over Valuation Rings*, J. Algebra **181**(1996), 836-856.

[G2] B.Green, *On Curves over Valuation Rings and Morphisms to* P^1, J. Number Theory **59**(1996), 262-290.

[Kna1] H.Knaf, *Divisors on varieties over valuation domains*, Israel J. Math **119**(2000), 349-377.

[Kna2] H.Knaf, *Regular Curves over Prüfer Rings*, Fields Institute Comm. **33**(2003), 89-105.