Hilbert's tenth problem for rational function fields over p-adic fields

Hilbert's tenth problem asked if there exists an algorithm that, given a polynomial over the integers in any number of variables, decides whether this polynomial has an integer solution. In 1970, Matiyasevich proved, following earlier work of Davis, Putnam and Robinson, that diophantine equations over the integers are undecidable. Undecidability has been proved for various other rings and fields. For rational function fields over a field K, undecidability has been reduced to giving a diophantine definition of the valuation ring at t in K(t). I will show how one can use quadratic forms to define the valuation in a diophantine way for rational function fields over p-adic fields. This is joint work with Jeroen Demeyer (University of Ghent).