Speaker:
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Title: Open problems in valuation theory of positive residue characteristic

Abstract:
I will discuss several open problems for valued fields of positive residue characteristic. Many of them are mentioned in the survey paper [1], but I will provide more background and give some updates. Some main topics will be:

1) Given a valued field, some of its maximal immediate extensions may have better model theoretic properties than others. For instance, this can be observed in [2]. If the valued field has a truncation closed embedding in a power series field, this may extend to some of its maximal immediate extensions, but not to others. Is there a connection between the two problems? What does the existence of truncation closed embeddings say about model theoretic properties? I will introduce the notion of *extremal valued field* (see [3]). Is every such field existentially closed in all (or some) of its maximal immediate extensions? I will also list other problems about extremal valued fields. It is important to investigate them, because for instance $\mathbb{F}_p((t))$ is extremal.

2) Under which additional conditions is a valued field K existentially closed in a function field F that admits a K-rational place P? Does this always hold after a finite constant extension $K'|K$? What can be said about the relation of P to K-rational discrete places of F? What can be said if P admits local uniformization? For background, see [4].

3) Is a valued field roughly deeply ramified if all of its extensions are independent defect fields? Definitions and background from [5] will be given in my talk.

4) The Henselian Rationality Theorem (see [6]) works over tame fields. To which extent does it also work over perfect fields, deeply ramified fields, or extremal fields of positive characteristic?

[1] Kuhlmann, F.-V.: *Model theory of tame valued fields and beyond: recent developments and open questions*, submitted; arXiv:2512.06386

[2] Kartas, K.: *Decidability via the tilting correspondence*, Algebra and Number Theory **18** (2024), 209–24

[3] Anscombe, S. – Kuhlmann, F.-V.: *Notes on extremal and tame valued fields*, J. Symb. Logic **81** (2016), 400–416

[4] Kuhlmann, F.-V.: *On places of algebraic function fields in arbitrary characteristic*, Advances in Math. **188** (2004), 399–424

[5] Kuhlmann, F.-V. – Rzepka, A.: *The valuation theory of deeply ramified fields and its connection with defect extensions*, Transactions Amer. Math. Soc. **376** (2023), 2693–2738

[6] Kuhlmann, F.-V.: *Elimination of Ramification II: Henselian Rationality*, Israel J. Math. **234** (2019), 927–958