

Speaker:

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Title: Aspects of higher ramification theory

Abstract:

Let $(L|K, v)$ be a Galois extension of valued fields. Higher ramification theory studies the extension from the ramification field $(L|K)^r$ of $(L|K, v)$ up to L . If the residue characteristic of (K, v) is 0, then this extension is trivial, but otherwise it contains all the bad ramification and the defects of $(L|K, v)$. The defect (also called ramification deficiency) is a major obstacle to the solution of problems like local uniformization and the model theory of valued fields in positive (residue) characteristic. Thus it is essential to study the defect, and the fact that $L|(L|K)^r$ is a tower of Galois extensions of degree $p = \text{char } Kv$ allows to reduce the study of the defect, and more generally, of wild ramification, to that of normal extensions of degree p .

In my talk, I will introduce the notions *ramification field*, *higher ramification group* and *ramification ideal*. Generalizing earlier work of Paulo Ribenboim, I will describe some methods of computing ramification ideals, and give some information we have so far about their relation to the defect.

Further, I will talk about the computation of Kähler differentials of Galois extensions $(L|K, v)$ of prime degree. They play a key role in an alternative proof in [3] (avoiding the use of almost ring theory) of a theorem of Gabber and Ramero that characterizes deeply ramified fields via Kähler differentials. The computation uses presentations of the valuation ring of L as a module of the valuation ring of K .

In [4] I show how suitable extensions $(L|K, v)$ of prime degree give rise to definable coarsenings of the valuation rings of L and K . In the case of Artin-Schreier and Kummer extensions with wild ramification one can also define the unique ramification ideal. The coarsenings of the valuation ring of L , their maximal ideals, and the ramification ideals can be used for the classification of defects and for the presentation of the Kähler differentials and their annihilators.

This is joint work with Dale Cutkosky and Anna Rzepka.

[1] Kuhlmann, F.-V. – Rzepka, A.: *The valuation theory of deeply ramified fields and its connection with defect extensions*, Transactions Amer. Math. Soc. **376** (2023), 2693–2738

[2] Cutkosky, S.D. – Kuhlmann, F.-V. – Rzepka, A.: *On the computation of Kähler differentials and characterizations of Galois extensions with independent defect*, Math. Nachrichten **298** (2025), 1549–1577

[3] Cutkosky, S.D. – Kuhlmann, F.-V.: *Kähler differentials of extensions of valuation rings and deeply ramified fields*, submitted; arxiv:2306.04967

[4] Kuhlmann, F.-V.: *On certain definable coarsenings of valuation rings and their applications*, submitted; arXiv:2512.06391